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Exploring 2D Ambisonics by Amplitudes and Phases

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ABSTRACT

This paper presents a spectral-like reformulation of two-dimensional ambisonics, enabling an alternative representation of the sound field in terms of amplitudes and phases. We situate this work within electroacoustic music creation, framing spatial audio as a synthesis problem. The amplitude-phase approach we propose allows for continuous manipulation and distortion of the sound field, starting from the ambisonic encoder. We hypothesise that it simplifies the representation, understanding, and creative manipulation of 2D ambisonics, thereby extending its use beyond encoded directional point sources.

1 Introduction

In the field of electroacoustic sound and music creation, we are interested in the synthesis of ambisonic sound fields, in order to open up new possibilities for composers to represent, manipulate, and control spatial audio. Current spatial practices in ambisonics [1] range from 'mainstream' spatialization processes based on the acoustic modeling of point sources in a reverberant environment [2] to more holistic approaches such as spatial sound synthesis and spatial sound processing [3, 4, 5]. We propose in this article to consider these two approaches as a continuum by reformulating the representation of two-dimensional ambisonics.

In 2D High-Order Ambisonics of order N [6, 7], the sound field according to the azimuthal direction θ is represented by g, a 2π-periodic angular function, expressed as a combination of circular harmonics Y_m between $-N$ and N, where:

$$Y_m(\theta) = \begin{cases} \sin(|m|\theta) & \text{for } m < 0 \\ \cos(m\theta) & \text{for } m \geq 0 \end{cases}$$

For instance, Figure 1 shows the harmonics (Y_m) at ambisonic order 2. The sound field is traditionally described using harmonic coefficients $(a_0, a_{-1}, a_1, \ldots, a_{-N}, a_N)$ applied to the harmonic functions (Y_m), i.e:

$$g(\theta) = a_0 Y_0(\theta) + \sum_{m=1}^{N} a_{-m} Y_{-m}(\theta) + a_m Y_m(\theta)$$

For example, when reading a 2D ambisonic file at order N (recorded before its decoding), each of its $(2N+$

1) tracks corresponds to the successive components $a_0, a_{-1}, a_1, \ldots, a_{-N}, a_N$.

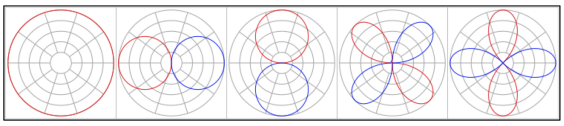


Fig. 1: Circular harmonics at ambisonic order 2.

If we set $b_m = a_{-m}$ to take the same notation as in Fourier's decomposition and replace the (Y_m) functions by their expressions in θ, the formula becomes:

$$g(\theta) = a_0 + \sum_{m=1}^{N} b_m \sin(m\theta) + a_m \cos(m\theta)$$

This corresponds to the Fourier synthesis[1] of the sound field as a function of the angle θ. In this formulation, the ambisonic order N then becomes the "spectrangular" resolution of this projection.

In this paper, we reformulate this Fourier representation in terms of amplitudes and phases, define its mathematical expression for the ambisonic encoder, and generalize it to other spatial configurations. Subsequently, we will show how to reformulate common spatial operations in ambisonics using amplitudes and phases.

2 Spectral-like reformulation

This Fourier representation can be reformulated in terms of amplitudes and phases, where each pair of harmonic coefficients (b_m, a_m) corresponds to an amplitude and a phase, similarly to Fourier's formalism. This leads to a compact description consisting of N+1 amplitudes (denoted c_m, except a_0 for the first one[2]) and N phases (denoted ϕ_m), combined in a vector $\hat{Y} = (a_0, c_1, \ldots, c_N, \phi_1, \phi_2, \ldots, \phi_N)$.

Every pair (b_m, a_m) for $m > 0$ is represented by its amplitude c_m and its phase ϕ_m[3], with:

$$\begin{cases} c_m = \sqrt{{b_m}^2 + {a_m}^2} \\ \phi_m = \frac{1}{2\pi} \arctan_2(b_m, a_m) \end{cases}$$

Conversely we have:

$$\begin{cases} a_m = c_m \cos(2\pi\phi_m) \\ b_m = c_m \sin(2\pi\phi_m) \end{cases}$$

This approach of the ambisonic sound field synthesis according to amplitudes and phases can be described as 'spectral ambisonics'. From the perspective of spatial synthesis, we choose to define this representation without sound (or as being applied a constant source at 1), as vectors of dimension $2N+1$ at the N^{th} order of ambisonics, to which operations can be applied. Sound will be added at the end of the process by convolution with the vector representing the constructed ambisonic situation, as shown in 5.3.2 subsection.

3 Reformulating the ambisonic encoder

The encoding process is generally the first step of an ambisonic process. It corresponds to the most directionnal spatial situation in ambisonics. Encoding a constant source equal to 1 in the angular direction θ is represented by the following harmonic coefficients of the encoding vector [8]:

$$Y(\theta) = (1, \sin(\theta), \cos(\theta), \ldots, \sin(N\theta), \cos(N\theta))$$

When computing the amplitudes, we get:

$$\forall k \in [1, N], c_k = \sqrt{\sin^2(k\theta) + \cos^2(k\theta)} = 1$$

For the phases, we have:

$$\forall k \in [1, N], \phi_k = k\frac{\theta}{2\pi}$$

Therefore, we get the following spectral-like reformulation[4] of the ambisonic encoder:

$$\hat{Y}(\theta) = (\underbrace{1, \ldots, 1}_{\text{N+1}}, \frac{\theta}{2\pi}, 2\frac{\theta}{2\pi}, \ldots, N\frac{\theta}{2\pi})$$

Only the phases are variable and all are proportional to $\frac{\theta}{2\pi}$. For phases, this representation of the encoder is identical to that of the frequencies of a harmonic sound, which are multiples of a given fundamental frequency. Therefore, the amplitude-phase approach can draw on previous contributions from the field of sound synthesis,

[1] In this paper, we do not deal with Fourier analysis of any $g(\theta)$ periodic angular function.

[2] Please note that a_0 can be positive or negative, as it is the harmonic coefficient of Y_0.

[3] In the music technology community, phases are typically normalized to $[0, 1]$.

[4] Denoted $\hat{Y}$, even in the rest of the paper, when the sound field is no longer an ambisonic encoder.

in which a harmonic sound can be progressively modified to make it inharmonic using spectral interpolation techniques introduced by composers John Chowning [9], Jean-Claude Risset [10], Tristan Murail [11], and Philippe Manoury [12] for spectral interpolation. This opens a vast field of exploration on distortion operations of the sound field starting from the ambisonic encoder.

4 Generalizing the encoder in spectral ambisonics

4.1 Interpretation of the $\hat{Y}$ vector

How can we interpret the $\hat{Y}$ vector of ambisonic spectral components in general? Let us decompose the contributions of amplitudes and phases.

- Changing the values of the amplitudes with all phases at 0 changes the shape of the sound field and its number of lobes. We illustrate this at third ambisonic order with two successive amplitude modifications: we start with $\hat{Y}_1 = (0.323, 0.189, 0.906, 0.417, 0, 0, 0)$ represented on the left of Figure 2 ; then we decrease a_0 to -1, which gives $\hat{Y}_2 = (-1, 0.189, 0.906, 0.417, 0, 0, 0)$ represented in the middle; finally we increase c_1 to 0.843 which gives $\hat{Y}_3 = (-1, 0.843, 0.906, 0.417, 0, 0, 0)$ shown on the right.
- Changing the phases without modifying the amplitudes does not change the number of lobes but causes continuous and periodic deformation of the ambisonic sound field. This is the result of the rotations of the Y_k (for $k > 0$) caused by changes of the various phases ϕ_k, mixing elementary positive and negative lobes. The main contribution comes from the phase ϕ_{k_0}, where c_{k_0} is the greatest amplitude among the (N+1) at ambisonic order N. Moving ϕ_{k_0} from 0 to 1 provokes a rotation and deformation of the sound field: some lobes enlarge and others shrink. This is what we show in Figure 3 starting from the ambisonic vector $\hat{Y}_1 = (0.323, 0.189, 0.906, 0.417, 0, 0, 0)$ on the top left. Here c_2 is the greatest amplitude, therefore we vary ϕ_2. The five other visualizations (from left to right and top to bottom) show the resulting ambisonic field when increasing ϕ_2, which takes five values: $\phi_2 = 0.134$, $\phi_2 = 0.299$, $\phi_2 = 0.630$, $\phi_2 = 0.843$, $\phi_2 = 1$.

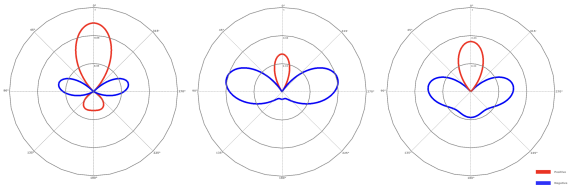


Fig. 2: Example of contribution of the amplitudes a_0 and c_1 to the ambisonic sound field.

To get a sense of this approach to amplitude and phase in 2D ambisonics, readers can use the interactive online visualizer[5] we have developed.

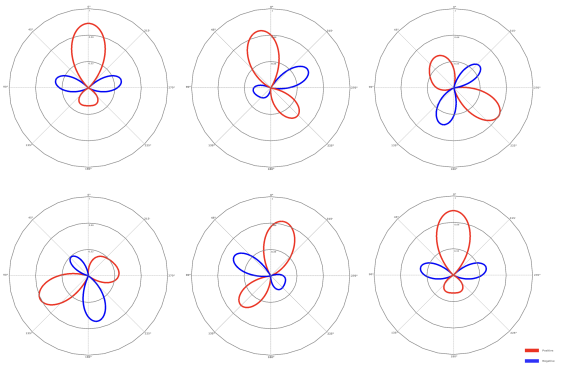


Fig. 3: Ambisonic sound field resulting from varying the phase ϕ_2 over $[0, 1]$.

4.2 From the encoder to a complex sound field

Among the synthesis techniques evoked in Section 3, Murail's approach [11] provides a simple model of deformation of an encoder. Murail proposes spectra based on sets of partials whose frequencies are expressed as

$$f_k = k^\alpha f \text{ for } k > 0$$

f being the fundamental frequency ($f > 0$) and α being any real number. When $\alpha = 1$ the spectrum is harmonic, when $\alpha \neq 1$ it becomes inharmonic. We can transpose this model to organize the phases of our spectral ambisonic vector $\hat{Y}$. By analogy, we set

$$\phi_k = k^\alpha \phi \text{ for } k > 0$$

ϕ being the 'fundamental' phase coming from the (Y_{-1}, Y_1) pair. As in the encoder, all the amplitudes

[5] Link to our Circular Harmonics Explorer: https://exploring2dambisonics.github.io/spectrangular_explorer.html

remain at 1. When $\alpha = 1$, the sound field corresponds to an encoder, when $\alpha \neq 1$ it is a complex sound field.

Continuously varying the value of α from 1 to a value greater than 1 (3 for instance) produces a rotation and distortion of the original encoder that proves compositionally interesting[6], adding liveliness and a sense of irregularity compared to the simple rotation of an encoder. Figure 4 shows the ambisonic sound field for various values of α: 1 (encoder), 1.5, 2 and 2.5, with $\phi = 0.25$.

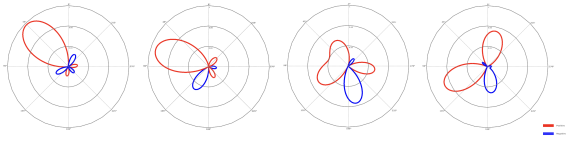


Fig. 4: From an encoder to a complex sound field, by varying the exponent α in Murail's model.

5 Spatial operations with spectral ambisonics

5.1 Convolution of two ambisonic sound fields

Let us consider two ambisonic situations at the same ambisonic order N described spectrally as $\hat{Y}_1 = (a_{1,0}, c_{1,1}, \ldots, c_{1,N}, \phi_{1,1}, \ldots, \phi_{1,N})$ and $\hat{Y}_2 = (a_{2,0}, c_{2,1}, \ldots, c_{2,N}, \phi_{2,1}, \ldots, \phi_{2,N})$. In analogy with the convolution of temporal signals when spectrally represented by amplitudes and phases, we introduce the spatial convolution of $\hat{Y}_1$ and $\hat{Y}_2$, defined by: $\hat{Y}_1 * \hat{Y}_2 = (a_{1,0} * a_{2,0},\ c_{1,1} * c_{2,1},\ \ldots,\ c_{1,N} * c_{2,N},\ \phi_{1,1} + \phi_{2,1},\ \ldots,\ \phi_{1,N} + \phi_{2,N})$.

Amplitudes are multiplied two by two, whereas phases are added two by two. We note that the convolution product of two ambisonic situations is itself an ambisonic situation. Moreover, thanks to the commutativity and associativity of addition and multiplication, the convolution product is itself commutative and associative.

We will now show that convolution is the central operation of 2D spectral ambisonics, enabling the representation of all common spatial operations on ambisonic sound fields, either by transformation of the phases or the amplitudes.

[6] Whereas values less than 1 result in only a very slight change.

For all the operations, we will consider an original ambisonic field $\hat{Y}_1 = (a_0, c_1, \ldots, c_N, \phi_1, \ldots, \phi_N)$ that we will convolve by a specific transformational vector $\hat{Y}_2$. We use the ambisonic sound field at order 3, with $\hat{Y}_1 = (0.055, 0.220, 0.882, 0.543, 0.661, 0.654, 0.417)$ for all examples and figures listed below.

5.2 Spatial operations on the phases

5.2.1 Rotation

The simplest application of convolution as defined above enables the rotation of an ambisonic field [8, 13]. Let us consider the convolution of an ambisonic sound field $\hat{Y}_1$ by an encoder at angle θ (represented by the phase $\phi = \frac{\theta}{2\pi}$). In this case, we have:

$$\hat{Y}_2 = (\underbrace{1, \ldots, 1}_{\text{N+1}}, \phi, 2\phi, \ldots, N\phi)$$

Therefore, we get:

$$\hat{Y}_1 * \hat{Y}_2 = (a_0, c_1, \ldots, c_N, \phi_1 + \phi, \phi_2 + 2\phi, \ldots, \phi_N + N\phi)$$

The amplitudes are not modified, but the phases are shifted by the phases of the encoder. A convolution by an ambisonic encoder at angle θ yields to the rotation of the sound field by an angle θ. Figure 5 shows $\hat{Y}_1$ on the left, an ambisonic encoder at angle $\frac{2\pi}{6}$ described by $\hat{Y}_2$ in the middle, and the rotated sound field on the field after the convolution $\hat{Y}_1 * \hat{Y}_2$.

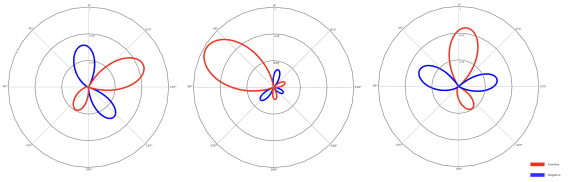


Fig. 5: An example of rotation of the ambisonic sound field by convolution with an encoder.

5.2.2 Symmetries

Mirror symmetry in ambisonics has already been formalized in geometric terms [14]. In 2D, symmetries include axial and point symmetries or reflections.

Horizontal symmetry The horizontal symmetry reverses the left and right parts of the ambisonic field: what was heard on the left is now heard on the right, and vice versa. This symmetry consists of multiplying the phases of the original ambisonic sound field by -1, which results in the vector $(a_0, c_1, \ldots, c_N, -\phi_1, \ldots, -\phi_N)$. This can be achieved by the convolution of $\hat{Y}_1$ by $\hat{Y}_2 = (1, 1, \ldots, 1, -2\phi_1, \ldots, -2\phi_N)$. Figure 6 shows the original sound field $\hat{Y}_1$ on the left and its horizontal symmetric on the right.

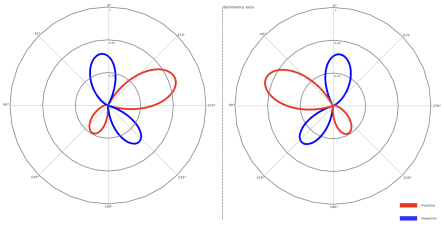


Fig. 6: An example of horizontal symmetry of the ambisonic sound field obtained by convolution.

Point symmetry Point symmetry around the origin is equivalent to a rotation of π, whose phase is $\frac{1}{2}$. Therefore $\hat{Y}_2 = (\underbrace{1, \ldots, 1}_{\text{N+1}}, \frac{1}{2}, \ldots, \frac{N}{2})$.

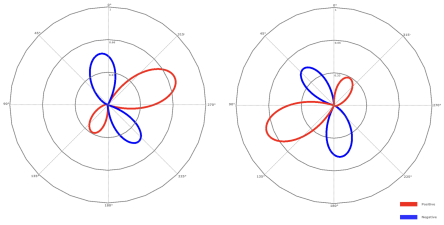


Fig. 7: An example of point symmetry of the ambisonic sound field obtained by convolution.

Vertical symmetry The vertical symmetry reverses the front and bottom parts of the ambisonic field: what was heard at the front is now heard at the bottom, and vice versa. The vertical symmetry is a combination of the two previous symmetries: a horizontal symmetry followed by a point symmetry. Therefore, it can be considered as the convolution of an ambisonic sound field $\hat{Y}_1 = (a_0, c_1, \ldots, c_N, \phi_1, \ldots, \phi_N)$ with the vector $\hat{Y}_2 = (1, 1, \ldots, 1, \frac{1}{2} - 2\phi_1, \ldots, \frac{k}{2} - 2\phi_k, \ldots, \frac{N}{2} - 2\phi_N)$.

Figure 8 shows the original sound field on the top and its vertical symmetric on the bottom.

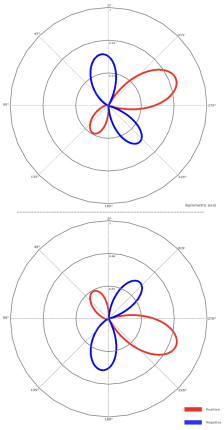


Fig. 8: An example of vertical symmetry on the ambisonic sound field by convolution.

5.3 Spatial operations on the amplitudes

5.3.1 General characteristics of amplitude operations

The operations we present here were initially designed [7] to be applied to ambisonic components $(a_0, b_1, a_1, \ldots, a_N, b_N)$. However, for all of them, the coefficients that multiply b_k and a_k are the same. This confirms our approach to consider the circular harmonics by pairs $(Y_{-k}(\theta), Y_k(\theta))$. Moreover, the approach in amplitudes and phases simplifies the computation, requiring the coefficients to be applied only once to each of the (N+1) amplitudes $(a_0, c_1, \ldots, c_N)$.

5.3.2 Applying a mono source by convolution

Applying a given mono sound $S(t)$ to a given ambisonic situation $(a_0, b_1, a_1, \ldots, b_N, a_N)$ gives $(a_0 S(t), b_1 S(t), a_1 S(t), \ldots, b_N S(t), a_N S(t))$. This can be achieved spectrally by convolution. If we spectrally represent the mono source $S(t)$ as:

$$\hat{Y}_1 = (\underbrace{S(t), S(t), \ldots, S(t)}_{\text{N+1}}, \underbrace{0, \ldots, 0}_{\text{N}})$$

and the existing sound field by $\hat{Y}_2 = (a_0, c_1, \ldots, c_N, \phi_1, \ldots, \phi_N)$, the application of the sound $S(t)$ to $\hat{Y}_2$ is spectrally represented by the convolution:

$$\hat{Y}_1 * \hat{Y}_2 = (a_0 S(t), c_1 S(t), \ldots, c_N S(t), \phi_1, \ldots, \phi_N).$$

5.3.3 Blurring

The spectral ambisonics framework naturally extends to filtering the amplitudes of a sound field. An example of filtering is blurring, which can be obtained by progressive fading of the amplitudes. Let us consider b, an adjustable degree of blurring between 0 and 1: if $b = 1$, the blurring is maximal, which means in terms of ambisonics that the sound field is reduced to the Y_0 harmonic, having no directionality.

Many functions may be used for fading; for instance, [7] proposes a vector of polynomial gains. Another convenient and efficient solution for blurring is the power function combined with a mapping function applied to b. Any decreasing easing function from $[0,1]$ to $[0,1]$ can be used for the mapping of b. For instance, we propose the circular function represented on $[0,1]$ in Figure 9:

$$m: \begin{array}{ccc} [0,1] & \longrightarrow & [0,1] \\ b & \longrightarrow & m(b) = 1 - \sqrt{b(2-b)} \end{array}$$

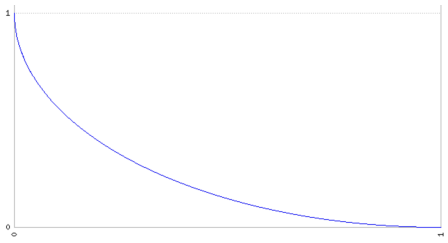


Fig. 9: The mapping function $m(b) = 1 - \sqrt{b(2-b)}$.

Starting from an ambisonic sound field $\hat{Y}_1 = (a_0, c_1, \ldots, c_N, \phi_1, \ldots, \phi_N)$, let us consider the vector:

$$\hat{Y}_2 = (1, m(b), m^2(b), \ldots, m^N(b), \underbrace{0, \ldots, 0}_{\text{N}})$$

The convolution of the two vectors $\hat{Y}_1 * \hat{Y}_2$ results in vector $(a_0, c_1 m(b), c_2 m^2(b), \ldots, c_N m^N(b), \phi_1, \phi_2, \ldots, \phi_N)$. We increasingly add blurring with increasing values of b: $b = 0$, $b = 0.2$, $b = 0.4$, $b = 0.6$, $b = 0.8$, $b = 1$ by computing the convolution product $\hat{Y}_1 * \hat{Y}_2$, as shown in Table 1. Blurring the ambisonic sound field can also be interpreted as continuously changing the ambisonic order from the original value (for instance $N = 3$ in our case) to 0 (when only Y_0 remains), that is from a directional to an omnidirectional restitution. This can be seen as going through fractional ambisonic orders [7].

5.3.4 Ambisonic optimization

Optimizing an ambisonic sound field can be achieved in two ways[7]: MaxRe or InPhase. MaxRe optimizes the energy vector and should be used for listeners confined in the center of the array of loudspeakers. InPhase [15] optimizes the energy vector and puts all loudspeaker signals in phase. Both can be interpreted as multiplications on the amplitudes of the original ambisonic sound field $\hat{Y}_1 = (a_0, c_1, \ldots, c_N, \phi_1, \ldots, \phi_N)$.

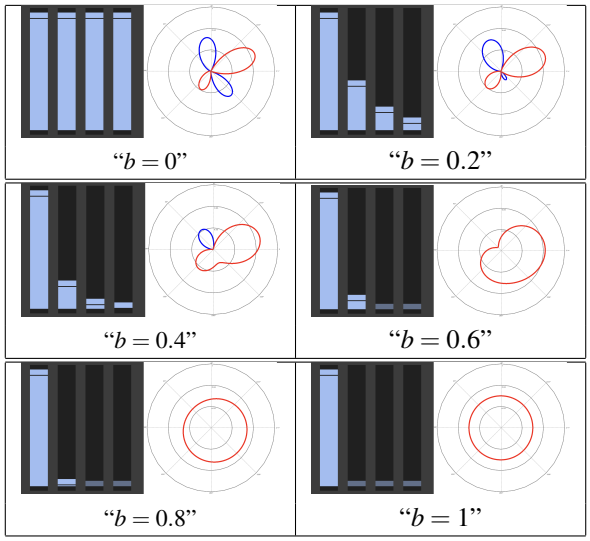


Table 1: Amplitudes (a_0, c_1, c_2, c_3) and ambisonic sound fields for various values of the blurring parameter b.

MaxRe optimization For MaxRe optimization at ambisonic order N, we define the vector:

$$\hat{Y}_2 = ((\cos(\frac{k\pi}{2N+1}))_{k=0}^{k=N}, 0, 0, \ldots, 0)$$

and the convolution product $\hat{Y}_1 * \hat{Y}_2$ provides the MaxRe optimization of the original ambisonic sound field $\hat{Y}_1$.

Figure 10 shows the ambisonic sound field $\hat{Y}_1$ on the left. We set $\hat{Y}_2 = ((\cos(\frac{k\pi}{7}))_{k=0}^{k=3}, 0, 0, 0) = (1, 0.900, 0.623, 0.223, 0, 0, 0)$. The convolution product $\hat{Y}_1 * \hat{Y}_2$ gives the MaxRe optimization of $\hat{Y}_1$ (on the right).

[7] The basic optimization, which does not affect the ambisonic sound field, is not discussed here.

InPhase optimization For InPhase optimization at ambisonic order N, we define the vector:

$$\hat{Y}_2 = ((\frac{(N!)^2}{(N-k)!(N+k)!})_{k=0}^{k=N}, 0, 0, \ldots, 0)$$

The (N+1) coefficients can be computed recursively, which will be more efficient, using the function $q(k,N)$:

$$q(k,N) = \begin{cases} 1 & \text{for } k = 0 \\ q(k-1,N)\frac{N-k+1}{N+k} & \text{for } k > 0 \end{cases}$$

Then the vector $\hat{Y}_2$ can be expressed as:

$$\hat{Y}_2 = ((q(k,N))_{k=0}^{k=N}, 0, 0, \ldots, 0).$$

Figure 10 shows the ambisonic sound field $\hat{Y}_1$ on the left. We set $\hat{Y}_2 = ((q(k,3))_{k=0}^{k=3}, 0, 0, 0) = (1, 0.75, 0.3, 0.05, 0, 0, 0)$. The convolution product $\hat{Y}_1 * \hat{Y}_2$ gives the InPhase optimization of $\hat{Y}_1$ represented on the right.

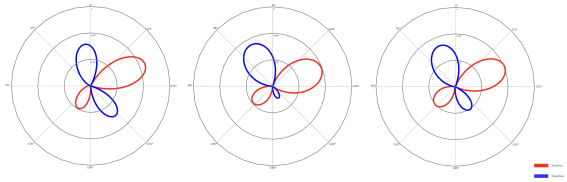


Fig. 10: An example of InPhase (center) and MaxRe (right) ambisonic optimization.

5.3.5 Widening

The widening of the sound field applies a 'fisheye' effect to the ambisonic components, controlled by a width value w varying on $[0,1]$. This can be defined as multiplying the amplitudes by various weights. The definition of the *weight* function, depending on the ambisonic order N, the index of the amplitude k and the width w, requires several steps and two auxiliary functions, *cw* and *c*. In the first step, we define:

$$weight(N,k,w) = (ln(N+1) * w + 1) * cw(N,k,w)$$

The second step consists of defining the *cw* function:

$$cw(N,k,w) = \begin{cases} 1 & \text{for } k = 0 \\ c(\frac{(1-w)*ln(N+1)-ln(k)}{ln(k+1)-ln(k)}) & \text{for } k > 0 \end{cases}$$

The third and last step involves defining the c function which is a kind of extended clipping from $\mathbb{R}$ to $[0,1]$:

$$c(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ x & \text{for } x \in]0,1] \\ 1 & \text{for } x > 1 \end{cases}$$

The convolution of the original ambisonic sound field $\hat{Y}_1$ by

$$\hat{Y}_2 = (weight(N,0,w), \ldots, weight(N,N,w), \underbrace{0, \ldots, 0}_{\text{N}})$$

results in the widening of the sound field. Figure 11 displays the widening of the $\hat{Y}_1$ sound field with various width values w: 0, 0.2, 0.4, 0.7.

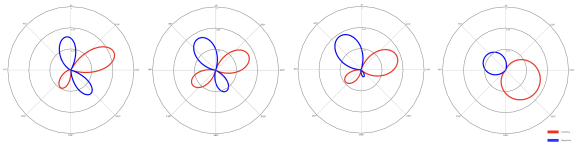


Fig. 11: Widening the sound field with various values of width w.

5.3.6 The location function

When synthetizing sound fields, the ambisonic model does not manage the distance of a sound source to the origin. A location function was introduced by [7] that simulates the placement of a sound source anywhere in the 2D plan located in polar coordinates (r_0, θ_0). For this purpose, we first need to set a distance-dependent attenuation function $a(r)$

$$a(r) = \begin{cases} 1 & \text{for } r \in [0,1[\\ \frac{1}{r^2} & \text{for } r > 1 \end{cases}$$

The second step involves encoding the sound source at angle θ_0 taking into account the attenuation function $a(r)$. This is described by the ambisonic vector:

$$\hat{Y}_1 = (\underbrace{a(r_0), a(r_0), \ldots, a(r_0)}_{\text{N+1}}, \frac{\theta_0}{2\pi}, 2\frac{\theta_0}{2\pi}, \ldots, N\frac{\theta_0}{2\pi}).$$

The third step consists of applying the widening vector defined in section 5.3.5, i.e:

$$\hat{Y}_2 = (weight(N,0,w_0),\ldots,weight(N,N,w_0),\underbrace{0,\ldots,0}_{\text{N}})$$

with $w_0 = max(1-r_0,0)$.

This leads to a large widening of the sound field around the origin. The location function is described by the convolution product $\hat{Y}_1 * \hat{Y}_2$.

5.4 Temporal processes

We can also apply temporal operations to the amplitude-phase sound field representation, as is commonly done for delays [7] or temporal decorrelation [16] on the circular harmonics. However, in this reformulation of the ambisonic field in amplitude-phase form, the rendering varies significantly depending on whether these operations are applied to the amplitudes, the phases, or both. Crucially, these temporal operations only manifest an effect once a spatial transformation is performed.

For instance, consider applying a set of delay lines by introducing a weighting factor on the delay duration as a function of the phase index k such that: $\Delta_k = d\frac{k}{N}$ where d is the delay duration and N is the order (equal to the number of phases). Furthermore, let us introduce time dependency to the encoding angle such that ϕ_k becomes $\phi_k(t) = k\frac{\theta(t)}{2\pi}$. In this context, the ambisonic sound field at order N is represented by the vector:

$$\hat{Y}(\theta,t) = (\underbrace{1,\ldots,1}_{\text{N+1}},\phi_1(t-\Delta_1),\phi_2(t-\Delta_2),\ldots,\phi_N(t-\Delta_N))$$

where $\phi_k(t-\Delta_k) = k\frac{\theta(t-k\frac{d}{N})}{2\pi}$

The processing yields a sonic impact only when the phases change and the particularity is that it doesn't impact the temporal morphology of the sonic source but rather its spatial morphology over a given duration. For example, changing the position of the projected source from one angle to another creates a jerking and stroboscopic trajectory, as shown in Figure 12.

5.5 Energy conservation

As defined by [17] and used in [16], the energy of an ambisonic sound field at order N represented by the vector $\hat{Y} = (a_0, c_1, \ldots, c_N, \phi_1, \ldots, \phi_N)$ is $E_{\hat{Y}} = a_0^2 + c_1^2 + \ldots + c_N^2$. Therefore, the ambisonic energy does not depend on the phases $\phi_1, \phi_2, \ldots, \phi_N$. This means that all the spatial operations on phases (rotations, symmetries) keep the ambisonic energy constant. Since it is defined as a function of the amplitudes, it is obvious that the energy of an ambisonic sound field will not remain constant when we apply operations on the amplitudes. However, it may be desirable to preserve the energy for the various spatial operations presented. As all the operations in this part will be processed using convolutions, we have to find a way to keep the energy of the result of a convolution product equal to the energy of the original ambisonic sound field. Therefore we are going to compute the energy of the result of a convolution and find a way to adjust it to the energy of the original sound field.

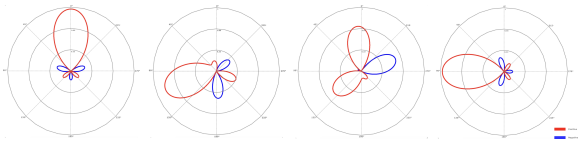


Fig. 12: $g(\theta)$ plot during a transition of θ from 0° to 90° ($d = 1000$ ms, $N = 3$). Panels from left to right correspond to $t - 1000$ ms, $t - 666$ ms, $t - 333$ ms, and t.

Let us consider two ambisonic situations at the same ambisonic order N described spectrally as $\hat{Y}_1 = (a_{1,0}, c_{1,1}, \ldots, c_{1,N}, \phi_{1,1}, \ldots, \phi_{1,N})$ and $\hat{Y}_2 = (a_{2,0}, c_{2,1}, \ldots, c_{2,N}, \phi_{2,1}, \ldots, \phi_{2,N})$. The convolution product is $\hat{Y}_1 * \hat{Y}_2 = (a_{1,0} * a_{2,0}, c_{1,1} * c_{2,1}, \ldots, c_{1,N} * c_{2,N}, \phi_{1,1} + \phi_{2,1}, \ldots, \phi_{1,N} + \phi_{2,N})$. Therefore we obtain $E_{\hat{Y}_1 * \hat{Y}_2} = (a_{0,1} * a_{0,2})^2 + (c_{1,1} * c_{1,2})^2 + \ldots + (c_{N,1} * c_{N,2})^2$.

If we want to keep the energy $E_{\hat{Y}_1}$ constant after convolution of $\hat{Y}_1$ with $\hat{Y}_2$, a solution consists of normalizing the amplitude of the vector $\hat{Y}_1 * \hat{Y}_2$ and multiply it by $\sqrt{E_{\hat{Y}_1}}$. Let us define $k = \frac{\sqrt{E_{\hat{Y}_1}}}{\sqrt{E_{\hat{Y}_1 * \hat{Y}_2}}}$. Since multiplying the amplitudes of an ambisonic vector by x results in multiplying the energy by x^2, we can define the ambisonic vector $\hat{Z}$ as:

$$\hat{Z} = (\underbrace{k,\ldots,k}_{\text{N+1}},\underbrace{0,\ldots,0}_{\text{N}})$$

and apply a double convolution $(\hat{Y}_1 * \hat{Y}_2) * \hat{Z}$. This vector has the same energy as $\hat{Y}_1$.

6 Conclusion

In this paper, we have introduced a spectral-like reformulation of the 2π-periodic angular function g representing the sound field in 2D High-Order Ambisonics. Since the g function is expressed as a combination of Y_m functions, it can be interpreted as a finite Fourier decomposition in terms of amplitudes and phases. This reformulation allows us to consider an ambisonic encoder by analogy with a harmonic sound, with all phases proportional to a 'fundamental' phase ϕ.Therefore, ambisonics in the amplitude-phase approach can draw on previous contributions from the field of sound synthesis, especially when interpolating between harmonic and inharmonic sounds. Taking the example of a model of synthesis proposed by Tristan Murail applied to spectral phases, we have shown that it can enable a continuous distortion of the original encoder. This opens new approaches of continuous interpolation between various approaches of spatialization in ambisonics, from point sources to macroscopic sound fields. We also introduced the concept of spectral convolution on amplitude-phase vectors, and we have shown just how central this concept is, enabling us to describe all usual spatial operations from rotation to source location. We also discussed temporal processes applied to the phases of the ambisonic sound field.

Our future work will focus on the following areas:

- An exploration of various sound synthesis models enabling interpolation from harmonic to inharmonic sounds applied to phases in spectral ambisonics.
- The design and development of an ambisonic synthesizer for electroacoustic composition, drawing inspiration from modular synthesizers and implementing the amplitude-phase models presented above.
- The evaluation of the approach presented in this paper by experimenting with this framework in our ambisonic studio, and later with our ambisonic synthetizer, inviting composers to assess our concept and our tools.
- An application of Fourier analysis enabling the decomposition of any periodic angular function $g(\theta)$ according $Y_m(\theta)$ functions, opening the scope of ambisonics and the research about spectral-like representation.
- The generalization of this spectral-like reformulation to 3D ambisonics.

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