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Estimating frequency-dependent absorption coefficients in small rooms using a diffusion model

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ABSTRACT

When modeling sound fields in rooms, information regarding the sound-absorbing properties of the room's surfaces is often required. Obtaining reliable boundary condition information can be a challenging task. Recently, the acoustic diffusion equation has been used in a numerical experiment to estimate a frequency-independent absorption coefficient. In this paper, we estimate frequency-dependent absorption coefficients between 20 Hz and 200 Hz from measured impulse response data of a small room. The estimated absorption coefficients are then used to predict room transfer functions, which are compared to measured data. Qualitative agreement between the data is found. This method could be used to infer room transfer functions in unmeasured regions of a room from a limited number of measured room impulse responses.

1 Introduction

Room acoustics predictions depend on the quality of the numerical model used and the input parameters of that model. Room acoustics models can be broadly categorized into wave, geometrical, and statistical models. These models vary in their levels of accuracy and computational cost. Once a suitable model has been chosen for a particular problem, the input parameters must be provided. Input parameters often include a description of the room's geometry, as well as source and boundary conditions. If the room is accessible, and the acoustic source is known *a priori*, it can be straightforward to define the geometry and source conditions. However, it can be difficult to supply reliable boundary conditions.

In this paper, we describe a method that uses an acoustic diffusion model to estimate the absorption coefficients of a small room's bounding surfaces at low frequencies. The acoustic diffusion equation is a geometrical model that was proposed by Picaut *et al.* [1]. The mathematical model describes non-uniform sound energy and energy flow within a room. The diffusion equation was numerically evaluated by Valeau *et al.* [2]. They presented source and boundary conditions and used the finite element method to study the diffusion equation when used to model cubic, long, and flat rooms. The numerical predictions agreed with statistical theory and with ray-tracing solutions. They noted that numerical solutions can be obtained efficiently, owing to a discretization resolution that is related to the mean free path of the room under consideration.

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Jing and Xiang [3] introduced a modified boundary condition for the diffusion equation. Finite element predictions obtained using the modified boundary condition were found to agree with measured data. Navarro *et al.* [4] used a finite difference implementation of the diffusion equation with the modified boundary condition to estimate an absorption coefficient in a numerical experiment. They assumed that the geometry and reverberation time of the room were known and subsequently solved an inverse problem to estimate a spatially uniform, frequency-independent absorption coefficient. They solved the diffusion equation in the time domain and estimated reverberation time from simulated impulse responses. More recently, researchers have been investigating the potential of the diffusion equation when used to estimate the absorption coefficients of materials in reverberation chambers using Bayesian parameter estimation [5, 6]. However, no experimental validation has been provided.

We use a finite element model of the diffusion equation to estimate spatially uniform, frequency-dependent absorption coefficients in a small room at low frequencies. These absorption coefficients are used to compute real-valued surface impedance values, which are, in turn, used as input parameters to a wave model of the Helmholtz equation. The predictions of the wave model are compared to measured data. In contrast to previous studies, an eigenvalue problem rather than a time domain problem is solved, and predictions are compared to measured data. This method offers the possibility to infer room transfer functions at remote, unmeasured receiver positions. This paper presents a step in this direction.

This paper is organized as follows: In Sec. 2, we introduce the diffusion equation, the reverberation time model, and the finite element-based inverse method used in this work. In Sec. 3, we introduce the room under test and present the models of the room's acoustics. In Sec. 4, we present the absorption coefficient estimates and compare the wave model predictions of the room transfer functions to the measured data. In Sec. 5, we discuss the possible applications and limitations of the proposed approach. We provide a summary in Sec. 6.

2 Inverse diffusion modeling

Valeau *et al.* [2] show that the diffusion equation can be understood as an extension of statistical diffuse field models, e.g., Sabine's model, to non-diffuse fields. We, therefore, assume that the low-frequency sound field in a small non-rectangular room can be modeled using the diffusion equation. In this section, we introduce the diffusion equation and the proposed absorption coefficient estimation method.

2.1 Diffusion equation

An acoustic diffusion equation is given by [1]

$$\frac{\partial w(t,\mathbf{x})}{\partial t} - D\nabla^2 w(t,\mathbf{x}) = 0\,, \tag{1}$$

where $w(t,\mathbf{x})$ is the sound energy density, which depends on time t and spatial position $\mathbf{x}$, and $D = \bar{l}c/3$ is the diffusion coefficient. Here, c is the speed of sound, and $\bar{l} = 4V/S$ is the mean free path given in terms of room volume, V, and surface area, S. Equation (1) can be supplemented with the modified boundary condition [3]

$$D\nabla w(t,\mathbf{x})\cdot\mathbf{n} = D\frac{\partial w(t,\mathbf{x})}{\partial n} = \frac{-c\,\alpha}{2(2-\alpha)}\,, \tag{2}$$

where $\mathbf{n}$ is the outwardly-pointing vector normal to a surface with $\|\mathbf{n}\| = 1$, $\partial/\partial n$ is the normal derivative, and α is an absorption coefficient. Assuming that the energy density follows an exponential decay law, we can express the sound energy density as [7, Eq. (5.6)]

$$w(t,\mathbf{x}) = w_0(\mathbf{x})\mathrm{e}^{-2\sigma t}\,, \tag{3}$$

with amplitude w_0 and damping coefficient σ. In preparation for eigenvalue analysis, we introduce the eigenvalue $\lambda = 2\sigma$.

2.2 Reverberation time and absorption coefficient estimates

With knowledge of the damping coefficients, the reverberation time can be computed using the formula [7, Eq. (3.70)]

$$T_{60} = \frac{3\ln(10)}{\langle\sigma\rangle}\,, \tag{4}$$

where $\langle\sigma\rangle$ is a frequency averaged damping coefficient. We chose to use third-octave frequency bands and introduce the average damping coefficient σ_j, and the reverberation time $T_{60,j}$, for the jth frequency band.

Using Sabine's formula [8], we can obtain an initial estimate of the jth absorption coefficient from a measurement of the reverberation time, as follows:

$$\hat{\alpha}_j = 0.161\frac{V}{ST_{60,j}}\,. \tag{5}$$

2.3 Finite element-based inverse method

With an initial estimate for $\hat{\alpha}_j$, we can use the diffusion equation to estimate an average damping coefficient for each frequency band, $\hat{\sigma}_j$. To do so, we generate a numerical model of the diffusion equation and write it as an eigenvalue problem. In this work, we choose to use the finite element method[1] to model Eq. (1).

Using the finite element method and Eqs. (2) and (3), we reformulate Eq. (1) to write the eigenvalue problem

$$\left[D\mathbf{K} + \frac{-c\,\alpha_j}{2(2-\alpha_j)}\mathbf{C} - \lambda\mathbf{M}\right]\mathbf{w}_0 = \mathbf{0}\,, \tag{6}$$

where $\mathbf{K}$, $\mathbf{C}$, and $\mathbf{M}$ are stiffness, damping, and mass matrices, respectively, and $\mathbf{w}_0$ is an eigenvector. We will not concern ourselves with the eigenvector. The elemental stiffness, damping, and mass matrices are given by

$$\mathbf{K}_e = \int_e \nabla\mathbf{N}^T \cdot \nabla\mathbf{N}\,\mathrm{d}V\,, \tag{7}$$

$$\mathbf{C}_{e'} = \int_{e'} \mathbf{N}^T\mathbf{N}\,\mathrm{d}S_\alpha\,, \tag{8}$$

$$\mathbf{M}_e = \frac{1}{c^2}\int_e \mathbf{N}^T\mathbf{N}\,\mathrm{d}V\,, \tag{9}$$

where the subscript e represents an interior element, e' represents a boundary element, S_α is a sound absorbing surface, and $\mathbf{N}$ are interpolating shape functions. The elemental matrices are solved numerically and used to assemble $\mathbf{K}$, $\mathbf{C}$, and $\mathbf{M}$.

Equation (6) does not explicitly depend on frequency. The frequency-dependency of the inverse method is manifested by the association of an absorption coefficient with a specific frequency or a band of frequencies. For a given absorption coefficient, the solution of Eq. (6) provides a set of eigenvalues. The smallest eigenvalue can be used to estimate $\hat{\sigma}_j$ [7, p. 101].

For each frequency band, the reverberation time is calculated from the measured data. Various reverberation time calculation methods can be found in the literature (e.g., [10]) - the approach used in this work is described in Sec. 3.3. Given the measured reverberation time and Eq. (4), we can then compute σ_j. The estimated absorption coefficient is found by solving the following optimization problem:

$$\tilde{\alpha}_j = \arg\min_{\hat{\alpha}_j} |\sigma_j - \hat{\sigma}_j(\hat{\alpha}_j)|. \tag{10}$$

In this work, we solve the problem iteratively, using $\hat{\alpha}_j$ from Eq. (5) as the initial value. As stopping criteria, both a predefined threshold and a maximum number of iterations are used. This process results in a set of absorption coefficient estimates that correspond to the third-octave band frequencies of interest.

3 Test case

Presented in this section are the room under test, the room impulse response measurement procedure, the reverberation time calculation, and the two numerical models of the room - the geometrical model used to estimate the eigenvalues of the diffusion equation and the wave model used to predict room transfer functions.

3.1 Room under test

A floor plan of the room considered in this work is shown in Fig. 1(a). The room has one slanted wall, a pillar, and two ventilation boxes on the ceiling, giving it a non-rectangular shape. The floor is carpeted, the walls are painted concrete, and the ceiling is made of plasterboard. The room has a volume of approximately 80 m^3 and a surface area of approximately 118 m^2.

3.2 Measurement setup and data

A low-frequency loudspeaker was placed on the floor in the bottom left-hand corner of the room. The room was excited with an exponential swept sine signal [11], and an omnidirectional microphone was used to capture the resulting sound field at multiple positions. In total, 2580 spatially dependent room impulse responses were recorded.

In addition to these measurements, two impulse responses in front of the loudspeaker driver were recorded. These measurements were separated by 4 cm on the axis of the driver and were used to estimate the axial velocity of the driver. This information is used to define the source condition in the wave model and is revisited in Sec. 3.4.2.

[1] For a detailed description of the finite element method when applied to room acoustics, see, e.g., Prinn [9].

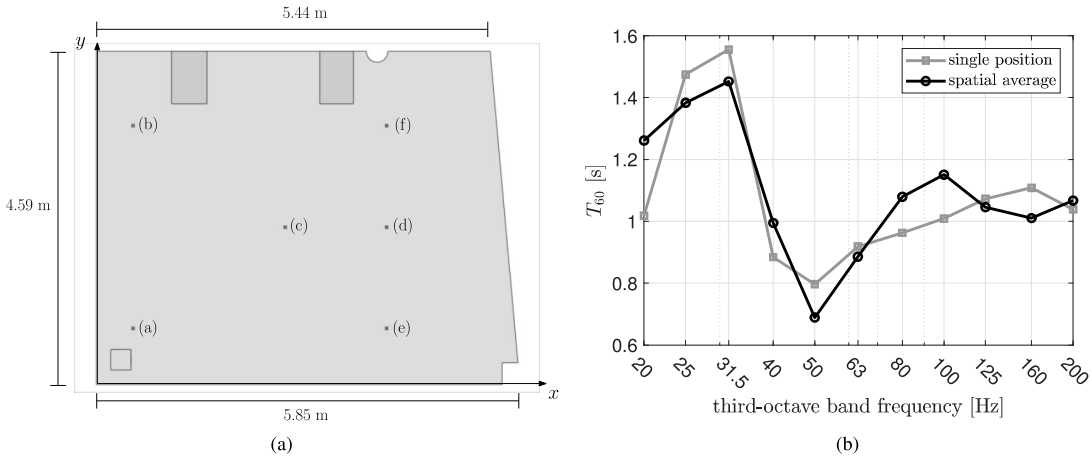


Fig. 1: Room under test. (a) Floor plan of the room. The source is located on the floor in the bottom-left corner of the room. The six points on the plan indicate microphone positions, at which the transfer functions are presented in Fig. 3. The room has a height of 3.13 m. (b) Reverberation time data of the room at microphone position (c) (with coordinates $\mathbf{x} = (2.61, 2.17, 1.07)$) and averaged over all 2580 positions.

3.3 Reverberation time calculation

To obtain the reverberation time values, the following procedure was used for each of the 2580 impulse responses:

1. a second-order Butterworth filter bank was used to generate a set of band-limited impulse responses - one for each third-octave band frequency in the range 20 Hz to 200 Hz,
2. for each band-limited impulse response, an energy decay curve was computed, using Schroeder's backward integration [12], and
3. the time taken for the energy to decay from - 5dB to -25 dB was multiplied by three to obtain the reverberation time.

While it is common to refer to these values as T_{20} values, for simplicity of notation, we refer to these values as $T_{60,j}$.

We consider two sets of frequency-dependent values in this work: the values obtained from a single measurement position (*single position*) and the averages of the values obtained at all measurement positions (*spatial average*). The reverberation times are shown in Fig. 1(b). The difference between the single position and the spatial average is clearly observed. This is a manifestation of the spatial non-uniformity of reverberation time at low frequency [13].

3.4 Computational room models

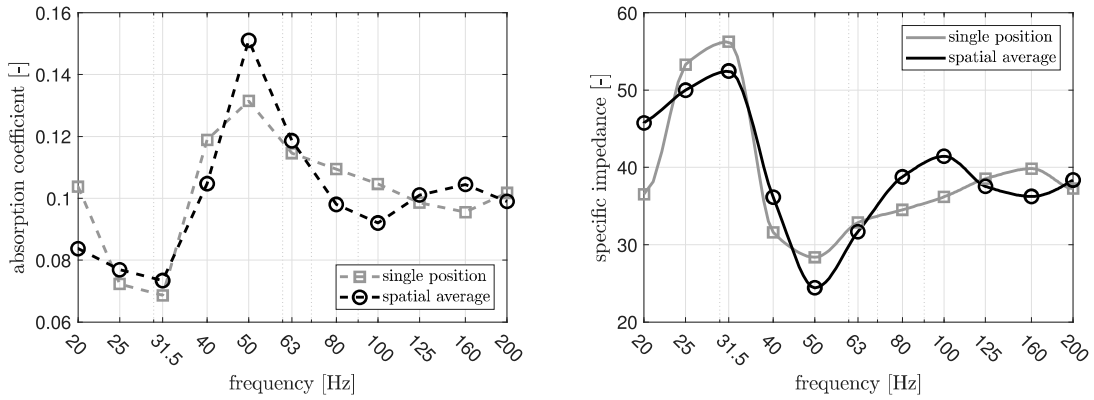
The room geometry was discretized into finite elements, and two models were generated: a geometrical model and a wave model.

3.4.1 Geometrical model

The geometrical model was used to estimate an absorption coefficient for each frequency band. We used the finite element method to discretize the diffusion equation with an element size of $\bar{l}/2$. This resolution provides highly efficient solutions. By solving the eigenvalue presented in Eq. (6), an estimate of the damping coefficient of the room's surfaces was found for a given absorption coefficient. Following a minimization of the difference between predicted and measured damping coefficients, estimates of the absorption coefficients were obtained (see Eq. 10).

3.4.2 Wave model

For validation without knowledge of the actual absorption coefficients, we compare predicted and measured



(a) (b)

Fig. 2: (a) Frequency-dependent absorption coefficient estimates. (b) Computed normal impedance curves.

room transfer functions. The predicted transfer functions were obtained using a finite element method description of the Helmholtz equation (see, e.g., [9]).

For a spatially uniform impedance, ζ, we can write

$$\left[\mathbf{K} + \frac{\mathrm{i}\omega}{c\zeta}\mathbf{C} - \frac{\omega^2}{c^2}\mathbf{M}\right]\mathbf{p} = \mathbf{q}\,, \tag{11}$$

where $\mathrm{i} = \sqrt{-1}$, ω is angular frequency, $\mathbf{p}$ is the unknown complex-valued pressure, and

$$\mathbf{q} = -\mathrm{i}\omega\rho \int_{e'} \mathbf{N}^T u_\mathrm{n}\,\mathrm{d}S_s\,, \tag{12}$$

is a source term, with medium density ρ, normal velocity u_n, and sound source surface S_s. The angular frequency is a free parameter that is independent of the frequency bands used for the estimation of the absorption coefficients.

The source term requires a description of the normal velocity, which, in this work, is approximated using the conservation of momentum as follows:

$$u_\mathrm{n} = \frac{\mathrm{i}}{\omega\rho}\frac{\partial p}{\partial n} \approx \frac{\mathrm{i}}{\omega\rho}\frac{p(\mathbf{x}_2) - p(\mathbf{x}_1)}{|\mathbf{x}_2 - \mathbf{x}_1|}\,. \tag{13}$$

In this approximation of the normal velocity, $p(\mathbf{x}_1)$ and $p(\mathbf{x}_2)$ are the room transfer functions obtained from the impulse responses measured at the two closely-spaced positions $\mathbf{x}_1$ and $\mathbf{x}_2$, which lie along the axis of, and are close to, the loudspeaker driver.

From the absorption coefficient estimates obtained using the geometrical model, we can estimate a real-valued impedance that serves as a boundary condition for the wave model. In this work, the frequency-dependent impedance values were computed using the formula:

$$\zeta_j = \frac{1 + \sqrt{1 - \hat{\alpha}_j}}{1 - \sqrt{1 - \hat{\alpha}_j}}\,. \tag{14}$$

Note that more elaborate impedance descriptions are available; see, e.g., Mondet *et al.* [14]. Interpolation was used to obtain a continuous description of ζ.

This wave model was used to validate the absorption coefficient estimates by predicting room transfer functions.

4 Results

This section presents the estimated absorption coefficients, computed impedance values, and comparisons of the measured and predicted room transfer functions.

4.1 Absorption coefficients

Estimated absorption coefficients for the single position reverberation time values and the spatially averaged reverberation time values are shown in Fig. 2(a). We observe that the absorption coefficient estimates are frequency-dependent, as expected. We can also see the effect of the different reverberation times. In general, the estimated absorption coefficient estimates are low, which indicates that this room is not heavily damped.

4.2 Real-valued normal impedance

The computed impedance values are shown in Fig. 2(b). We see that a higher absorption coefficient translates into a lower impedance value. Piecewise cubic interpolation is used to generate a smooth impedance function. The interpolated impedance is used as an input to the wave model, which is used to predict the room transfer functions.

4.3 Room transfer functions

The measured and predicted room transfer functions at six receiver positions are presented in Fig. 3. The positions are taken on one horizontal plane in the room, at approximately the ear height of a seated listener. The positions correspond to the points shown on the floor plan presented in Fig. 1(a). Note that un-calibrated measurement data has been used; thus, for comparison, the data are normalized by the sound pressure level at the first resonant peak of the room transfer function.

In general, both numerical solution sets (i.e., predicted transfer functions) capture the trend of the measured data well. Furthermore, many of the Q-factors of the resonant peaks are well represented. We also observe small differences between the predicted transfer functions obtained using the single position and those obtained using the spatial average reverberation times. The mean of the differences between the sound pressure levels of the two predicted room transfer functions is approximately 0.5 dB for all six positions.

5 Discussion

The transfer functions presented in Fig. 3 are encouraging. However, a discussion of the assumptions and practical limitations of this approach is required.

5.1 Assumptions and limitations

The shapes of the resonant peaks in the room transfer functions are well captured by the wave model. Thus, the inverse method has provided reasonable descriptions of this small room's absorbing surfaces at low frequencies. There are, however, discrepancies between the transfer functions. In reality, the room under test has spatially non-uniform boundary conditions. It is possible that the estimation of spatially uniform, real-valued impedances is not sufficient. Estimating spatially non-uniform real-valued impedances is left for future studies. While methods for estimating complex-valued impedances can be found in the literature (e.g., [15]), the method proposed here is efficient and, therefore, more applicable to audio applications.

We have provided the inverse method with initial estimates computed by Sabine's diffuse field model. Note that using alternative initial estimates, e.g., $\hat{\alpha}_j = 0.1$, does not affect the observed agreement but can result in slightly longer computation times. Also, as evidenced by comparing the two simulated data sets, small changes to the absorption coefficients do not appear to translate into large changes in the transfer functions. The room used here has relatively hard boundaries, as evidenced by the reverberation times. Additionally, there is no furniture in the room, and the room has no windows. It could be that this approach is less applicable when materials with high damping are present in the room under test. More testing is required before any firm conclusions can be drawn.

The inverse method is not limited to low frequencies and can, in theory, be used to estimate absorption coefficients for the entire audible range. We have limited our study to low frequencies for two reasons. The first reason is to make the comparisons of the data simpler. As the frequency increases, the number of modes will increase, and it will become increasingly difficult to confirm that the trend of the data has been captured by the estimated absorption coefficients. The second reason is that there is currently no method that can accurately simulate the sound field generated by a directional sound source in a complex-shaped room at high frequencies, see, e.g., Brinkmann *et al.* [16]. Therefore, a comparison of measured and simulated transfer functions obtained using absorption coefficients estimated at high frequencies is left for future work.

5.2 Possible applications

After taking the limitations into account, this approach could be used to infer the room transfer functions at low frequencies. This could be useful for audio equalization or possibly auralization applications. For the method to be more general, however, one would need to find a way to estimate the absorption coefficients of multiple surfaces for a given frequency range.

Additionally, given that obtaining absorption coefficient data in situ is challenging, this inverse method might be useful in collecting training and testing data for efficient data-driven approaches designed to estimate absorption coefficients.

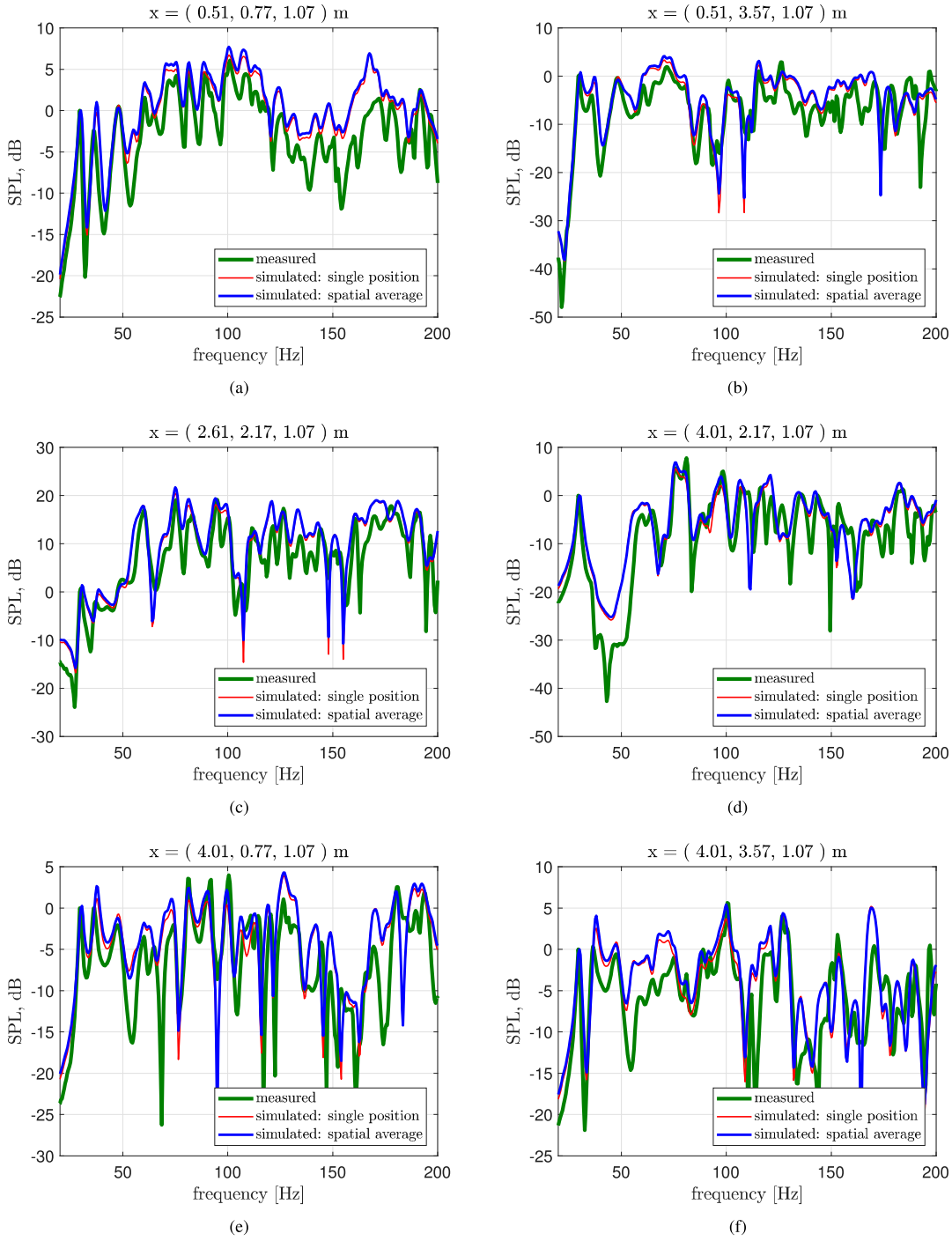


Fig. 3: Comparisons of measured and predicted room transfer functions. The measurement positions are indicated at the top of each panel. For each measurement position, the transfer functions have been normalized to the first resonant peak.

6 Summary

An inverse method based on the acoustic diffusion equation has been used to estimate the frequency-dependent absorption coefficients in a small non-rectangular room. The absorption coefficients have been used to estimate real-valued impedances, which have been used with the known room geometry, source position, and source description to predict the room transfer functions at low frequencies. The predicted transfer functions agree, at least qualitatively, with the measured data. The approach can be used to predict the sound field in a room at low frequencies.

Future work should focus on estimating non-uniform absorption coefficients and testing the method on complex-shaped, highly damped rooms.

7 ACKNOWLEDGMENT

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